Homework 1 solutions

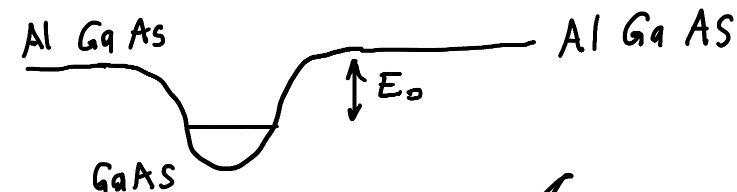
(1)

$$E_F = \hbar (3\pi^2 n)^{\frac{1}{3}} \approx 9.7 \text{ eV}$$

$$p_F = (2m^* E_F)^{\frac{1}{2}} \approx 1.5 \cdot 10^{-19} \frac{g \cdot cm}{s}$$

$$v_F = \frac{p_F}{m^*} \approx 2.1 \cdot 10^{8} \frac{cm}{s}$$

(2)



The total energy of quasiparticles consists of those of transverse and longitudinal motion $E = -E_0 + \frac{k^2}{2m^*}$ (if measured from the potential barrier in Al Ga As)

We require $E < 0$ for electron states to be confined to this 1D wire

In principle, there may be electrons propagating along the wire with higher energies, but they will get scattered in the transverse direction by imperfections.

So, the maximum longitudinal momentum is $k_F = (2m^* E_0)^{\frac{1}{2}}$

The 1D density of el-s

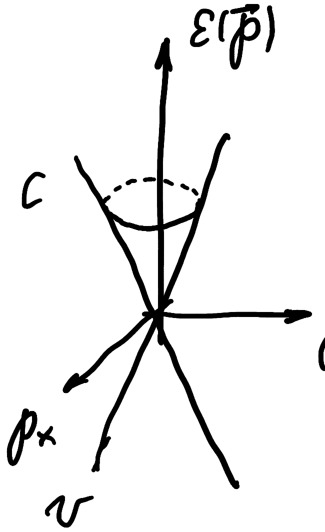
The 1D density of

$$n = 2\int_{-k_F}^{k_F} \frac{dk_z}{2\pi} = \frac{2k_F}{\pi} = \frac{2(2m^*E_0)^{\frac{1}{2}}}{\pi}$$

Recovering $\hbar$, $n = \dfrac{2(2m^*E_0)^{\frac{1}{2}}}{\pi\hbar} = 1.93\cdot 10^{6}\,\text{cm}^{-1}$

③

Heat capacity of graphene



Graphene has 2 quasiparticles for each momentum $\vec{p}$: in the conduction band (C) and in the valence band (V)

(We do not consiger spin here)

$\varepsilon(\vec{p}) = \pm v p$, as you may find by diagonalising the Hamiltonian

Particle excitations in the conduction band are equivalent to hole excitations in the valence band, so it's enough to consider only one of them

$$E = 2S\int \frac{d^2p}{(2\pi\hbar)^2}\, \varepsilon_p\, f(\varepsilon_p) =$$

$$= 2S\int \frac{2\pi p\, dp}{(2\pi\hbar)^2} \frac{vp}{e^{\frac{pv}{T}}+1} = \frac{S}{\pi\hbar^2}\frac{T^3}{v^2} \underbrace{\int_0^\infty \frac{x^2\,dx}{e^x+1}} =$$

$$= 2S \int \frac{\cdots}{(2\pi\hbar)^2} \frac{\cdots}{e^{\frac{pv}{T}}+1} = \frac{\cdots}{\pi\hbar^2 v^2} \underbrace{\int_0 \frac{\cdots}{e^x+1}}_{=\frac{3}{2}\zeta(3)}$$

$$= \frac{3\zeta(3)\, S}{2\pi\, \hbar^2 v^2} T^3$$, S – area of the graphene sheet

$$\boxed{C = \frac{9\zeta(3)\, S}{2\pi\, \hbar^2 v^2} T^2}$$